6.1 - Exponential Growth and Decay Functions

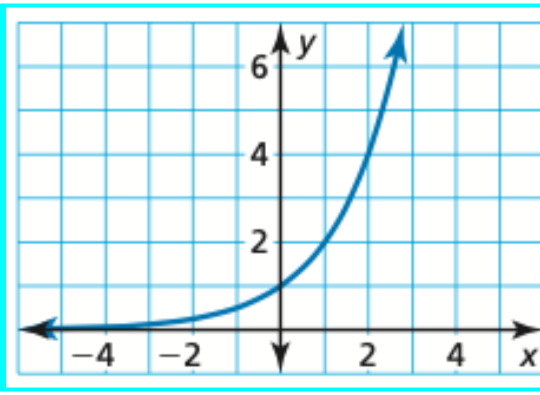
Exponential Function

Has the form $y = ab^x$ where $a \neq 0$ and base b is a positive real number other than 1.

Exponential Growth Function

When $a > 0$ and $b > 1$

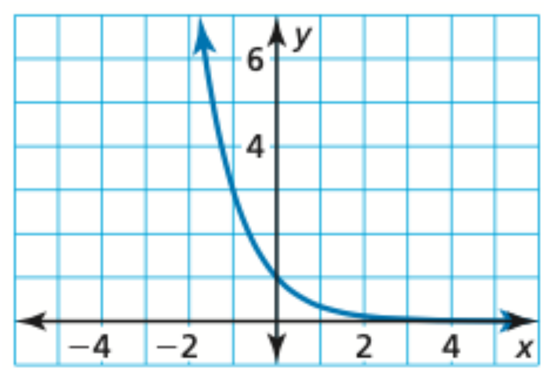
b is called the **growth factor**.



Exponential Decay Function

When $a > 0$ and $0 < b < 1$

b is called the **decay factor**.

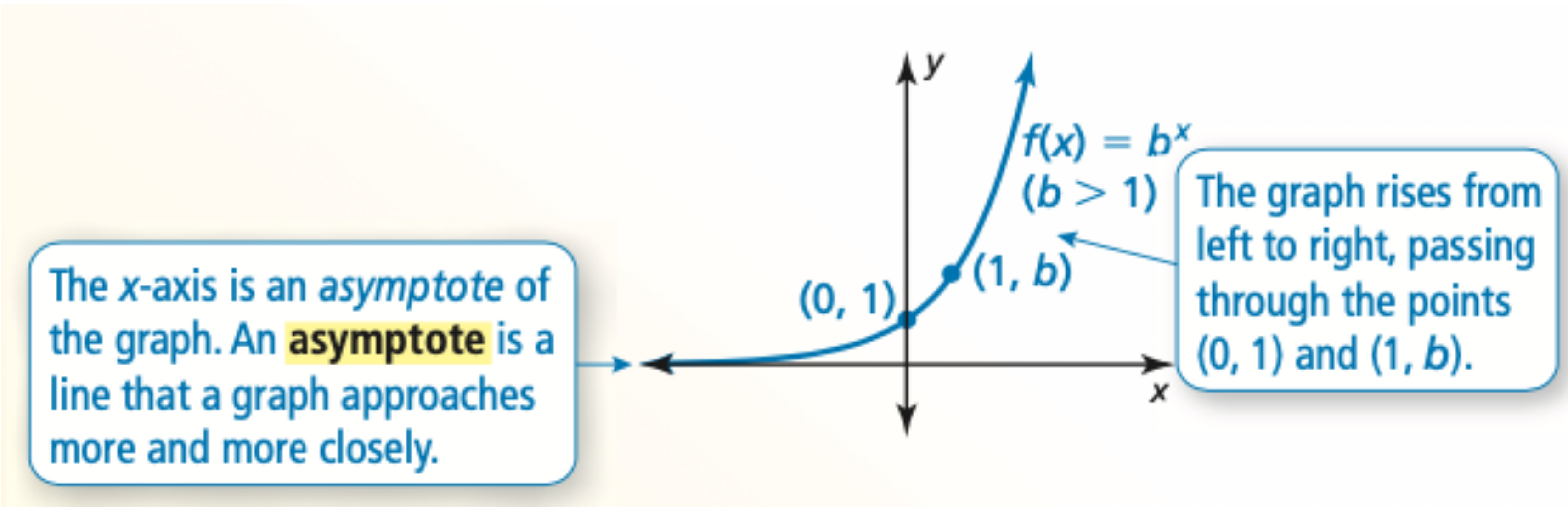


6.1 - Exponential Growth and Decay Functions

Parent Function for Exponential Growth

The function $f(x) = b^x$ where $b > 1$

The domain is all real numbers. Range is $y > 0$

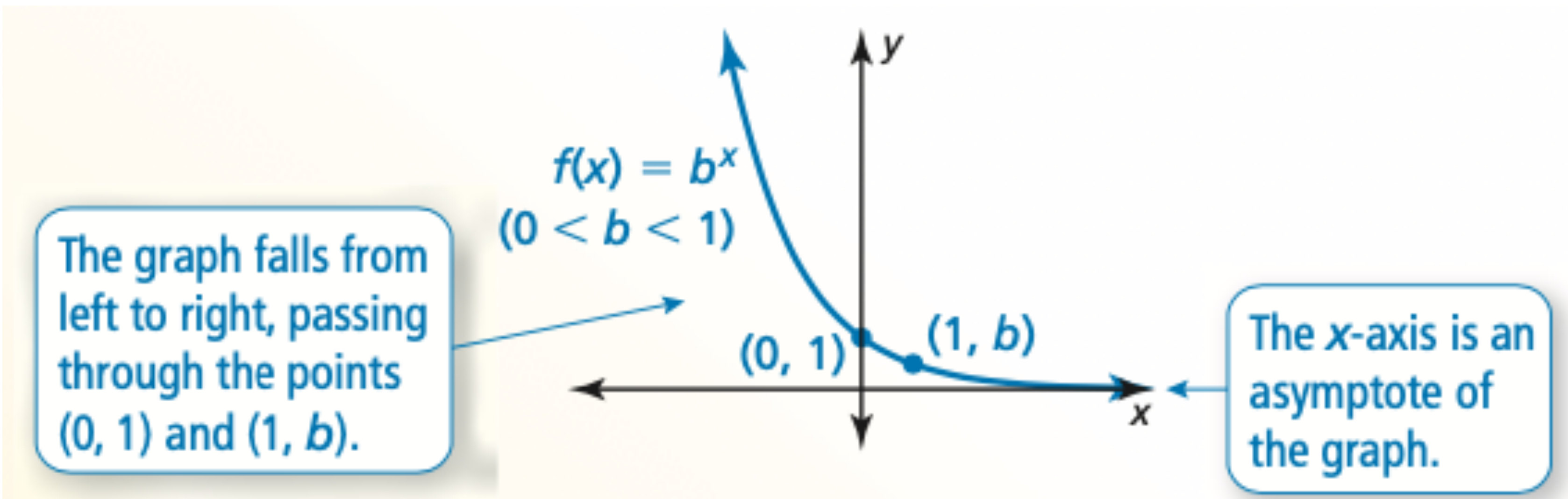


$$f(x) = 2^x$$

6.1 - Exponential Growth and Decay Functions

Parent Function for Exponential Decay

The function $f(x) = b^x$ where $0 < b < 1$
The domain is all real numbers. Range is $y > 0$



$$f(x) = \left(\frac{1}{2}\right)^x$$

6.1 - Exponential Growth and Decay Functions

Exponential Functions

$f(x) = ab^x$ $\quad b > 0$

What if? $f(x) = 3(-2)^x$

$f(0) = 3$ $\quad f(2) = 12$ $\quad$ Find $f(x)$

$f(x) = 3(2)^x$

Practice

1. $f(0) = 5$
   $f(3) = 40$

   $f(x) = 5(2)^x$

2. $f(0) = 80$
   $f(4) = 5$

   $f(x) = 80\left(\frac{1}{2}\right)^x$

6.1 - Exponential Growth and Decay Functions

Exponential Functions

x	g(x)	
-2	$\frac{4}{9}$	⟵
-1	$\frac{4}{3}$	
0	4	
1	12	
2	36	⟵

$$g(x) = ab^x$$

$$\frac{4}{9} = ab^{-2}$$

$$36 = ab^2$$

divide

$$\frac{1}{81} = b^{-4}$$

$$b = 3$$

$$\frac{4}{9} = a(3)^{-2}$$

$$a = 4$$

$$g(x) = 4(3)^x$$

6.1 - Exponential Growth and Decay Functions

Exponential Functions

x	h(x)
-2	128
2	$\frac{1}{2}$

$$h(x) = ab^x$$

$$h(x) = 8\left(\frac{1}{4}\right)^x$$

6.4 - Transformations of Exponential and Logarithmic Functions

Transformations

$$f(x) = 3^x$$

How do you transform $f(x)$ to obtain $h(x)$?

$a.\ h(x) = 3^{x+2}$

Translate left 2

$b.\ h(x) = 4(3)^x$

vertical stretch 4

6.4 - Transformations of Exponential and Logarithmic Functions

Transformations

$$f(x) = 2^x$$

How do you transform $f(x)$ to obtain $h(x)$?

$a.\ h(x) = 2^x + 3$

Translate up 3

$b.\ h(x) = 2^{3x}$

Horizontal shrink 1/3

$$c.\ h(x) = \left(\frac{1}{4}\right)^x + 7$$

$$= 2^{-2x} + 7$$

Reflect y-axis
horz. shrink 1/2
translate up 7

$$d.\ h(x) = \left(\frac{\sqrt{2}}{4}\right)^{-x}$$

$$= 2^{3x/2}$$

Horz. shrink 2/3

6.2 - The Natural Base e

Euler's Constant

Called the <u>natural base</u> and denoted by *e*.

Example: Compounded Interest

$R = 12\,\%$ Interest compounded monthly

$P = \$100$ Principal

$FV = 100\left(1 + \frac{0.12}{12}\right)^{12}$ Future value after 12 months

What if compounded daily? Hourly? …

$$\lim_{n\to\infty}\left(1 + \frac{1}{n}\right)^{n} = e = 2.71828...$$

6.2 - The Natural Base e

Euler's Constant

Called the natural base and denoted by *e*.

The actual value is $e = 2.71828182846...$

e can be calculated by the expression $(1+\frac{1}{x})^x$ as x approaches infinity.

x	10^1	10^2	10^3	10^4	10^5	10^6
$\left(1+\frac{1}{x}\right)^x$	2.59374	2.70481	2.71692	2.71815	2.71827	2.71828

6.3 - Logarithms and Logarithmic Functions

Logarithmic Functions

$$3^5 = 3^x$$
$$x = 5$$

$$log_3 3^5 = log_3 3^x$$
$$5 = x$$

$$log_{\frac{1}{3}}\left(\frac{\sqrt{3}}{9}\right) = x$$

$$\left(\frac{1}{3}\right)^x = \frac{\sqrt{3}}{9}$$

$$3^{-x} = 3^{-1.5}$$

$$x = 1.5$$

Practice

1. $\log_3 81$

   $x = 4$

2. $\log_2 \frac{1}{16}$

   $x = -4$

3. $\log_4 32$

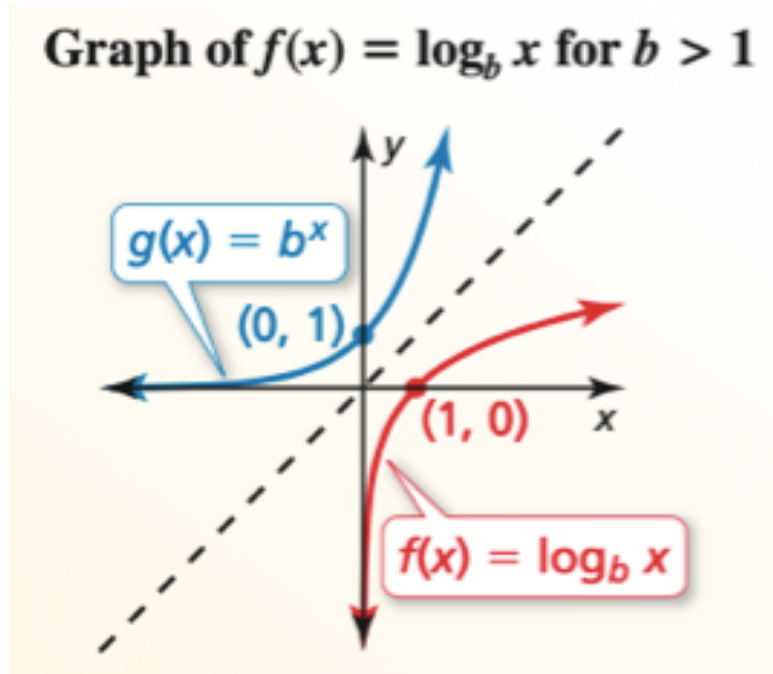
   $x = \frac{5}{2}$

6.3 - Logarithms and Logarithmic Functions

Graphing Logarithmic Functions

$$g(x) = 10^x$$

$$g^{-1}(x) = f(x) = \log_{10} x$$



What must be positive?

$$y = \log_b a$$

y is all real numbers

$$a > 0$$

$$b > 0$$

6.3 - Logarithms and Logarithmic Functions

Solving Logarithmic Functions

$\log_3 x = 2$ $\quad$ $3^2 = x$, $x = 9$

$\log_x 16 = 2$ $\quad$ $x^2 = 16$, $x = 4, \cancel{-4}$

Practice

1. $\log_6 x = 2$

   $x = 36$

2. $\log_x \dfrac{1}{9} = -2$

   $x = 3$

3. $\log_4 x = -\dfrac{3}{2}$

   $x = \dfrac{1}{8}$

What about? $x = \log_{\sqrt{2}}\left(\dfrac{\sqrt{2}}{32}\right)$

$x = -9$